UNIT-1

1. What is linear programming? Explain the graphical method of solving an LP Problem?

Linear programming:

Linear programming is a widely used mathematical modelling technique to determine the optimum allocation of scarce resources among competing demands. Resources typically include raw materials, manpower, machinery, time, money and space.

What is Linear programming:

LPP is a mathematical technique for maximizing (or) minimizing a linear function of several variables such as profit (or) cost.

Graphical Method:

Step 1: Convert the inequality constraint as equations and find co-ordinates of the line.

Step 2: Plot the lines on the graph. (Note: If the constraint is ≥ type, then the solution zone lies away from the centre.

If the constraint is ≤ type, then solution zone is towards the centre.)

Step 3: Obtain the feasible zone.

Step 4: Find the co-ordinates of the objectives function (profit line) and plot it on the graph representing it with a dotted line.

Step 5: Locate the solution point. (Note: If the given problem is maximization, z max then locate the solution point at the far most point of the feasible zone from the origin and if minimization, Z min then locate the solution at the shortest point of the solution zone from the origin).

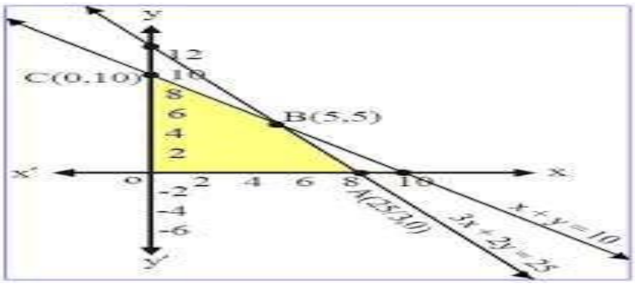
Step 6: Solution type

i. If the solution point is a single point on the line, take the corresponding values of x1 and x2.

ii. If the solution point lies at the intersection of two equations, then solve for x1 and x2 using the two equations.

iii. If the solution appears as a small line, then a multiple solution exists.

iv. If the solution has no confined boundary, the solution is said to be an unbound solution.



What is Unbounded Solution?

Unbounded Solution:

When determining the leaving variable of any tableau, if there is no positive minimum ratio or all entries of pivot column are negative or zero.

For example, consider the problem

Max $z = 2x_1 + x_2$; Subject to : $-x_1 + x_2 \leq 10$; $-2x_1 \leq 40$; $x_1, x_2 \geq 0$

X1 is the entering variable but all the constraint coefficients under X1 are ≤ 0, meaning that X1 can be increased indefinitely, which gives unbounded solution.

Basic Variable	x_1	x_2	S_1	S_2	Solution	Min Ratio
Z_j	-2	-1	0	0	0	-
S_1	-1	1	1	0	10	-
S_2	-2	0	0	1	40	-

UNIT-2

2. What is Transportation problem? Explain any two methods of Initial Basic Feasible Solutions?

It is a special kind of LPP in which goods are transported from set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively, such that to minimize the transportation cost and maximize the profits.

Types of Transportation problem:

1. Balanced transportation problems (S = D)

If the sum of the supplies of all the sources is equal to the sum of the demands of all the destinations, then the problem is termed as balanced transportation problem.

It may be represented by the relation: **Total Supply = Total Demand.**

2. Un Balanced transportation problem (S ≠ D)

If the sum of the supplies of all the sources is not equal to the sum of the demands of all the destinations, then the problem is termed as unbalanced transportation problem.

That means, for any unbalanced transportation problem, we have **Total Supply ≠ Total Demand.**

Initial Basic Feasible Solutions:

- *There are many ways to find a feasible solution. We will examine several below. The first is simple but ineffective, and we will then look at more complex but effective (producing near optimal solutions) methods.*

1. North West Corner Cell Method (NWCM)
2. Least Cost Cell Method (LCM)
3. VAM (Vogel's Approximation Method)

North West Corner Cell Method (NWCM):

- First we have check whether the given transportation problem is **balanced or not.**
- Balanced transportation problem then we have find out the **initial basic feasible solution.** other wise

- It has **Un balanced transportation problem** we have convert into the **un balanced transportation problem** into **balanced transportation problem.**
- It has **added** either **dummy row** or **dummy column** after we have find out initial basic feasible solution.

Least Cost Cell Method (LCM):

Step: - 1

Find the minimum of the (undeleted) values in the cost matrix (i.e. find the matrix minimum).

Step: - 2

Find the minimum of the supply and demand values (x) with respect to the cell corresponding to the matrix minimum.

Step: - 3

Allocate x units to the cell with the matrix minimum. Also, subtract x units from the supply and the demand values corresponding to the cell with the matrix minimum.

Step: - 4

Check whether exactly one of the row/column corresponding to the cell with the matrix minimum has zero supply/zero demand, respectively. If yes go to step-5 otherwise, go to step-6.

Step: - 5

Delete that row/column with respect to the cell with the matrix minimum which has the zero supply/zero demand and go to step-7.

Step: - 6

Delete the both row and column with respect to the cell with the matrix minimum.

Step: - 7

Check whether exactly one row or column is left out. if yes go to step-8 otherwise, go to step-1

Step: - 8

Match the supply/demand of that row/column with the remaining demands/supplies of the undeleted columns/rows.

Step: - 9

Go to phase 2.

3. Explain the procedure of Hungarian Method in solving an Assignment Problem?

Assignment Problem:

It is a special kind of transportation problem in which each source should have the capacity of the destination.

Procedure of Hungarian Assignment Method:

Step 1: Locate the smallest cost element in each row of the cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of the new table, called the Reduced cost table.

Step 2: In this reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest value from every other entry in the column. As a result, there would be at least one zero in each of the rows and columns of the second reduced cost table.

Step 3: Draw the minimum number of horizontal and vertical lines that are required to cover all the ‘Zero’. If the number of lines drawn is equal to n (the number of rows/columns) the solution is optimal, and proceeds to step 6. If the number of lines drawn is smaller than n, go to step 4.

Step 4: Select the smallest uncovered (by the lines) cost element.

Subtract this element from all uncovered elements including itself and add this element to each value located at the intersection of any two lines.

Step 5: Repeat steps 3 and 4 until an optimal solution is obtained.

Step 6: Make the job assignments to ‘Zero’ elements.

a) Locate a row which contains only one ‘zero’ element. Assign the job corresponding to this element. Cross out the zeros, if any, in the column corresponding to the element

b) Repeat (a) for each of such rows which contain only one zero.

Similarly, perform the same operation in respect of each column containing only one ‘zero’, crossing out the zero, if any, in the row

c) If there is no row or column with only a single ‘Zero’ left, then select a row/column arbitrarily and choose one of the jobs and make the assignment. Now cross the remaining zeros in the column and row in respect of which the assignment is made.

d) Repeat steps (a) through (c) until all assignments are made.

e) Determine the total cost with reference to the original cost table

UNIT-3

Dynamic programming:

The mathematical technique of optimizing a sequence of inter-related decisions over a period of time is called dynamic programming.

Bellman's Principle of Optimality:

Bellman's principle of optimality is the principle which guides the solution of a problem using dynamic programming.

It states:

Regardless of the decisions taken to enter a particular state in a particular stage, the remaining decisions made for leaving that stage must constitute an optimal policy"

Steps in Dynamic Programming:

1. Divide the original problem into sub-problems called stages.
2. Solve the last stage of the problem for all possible conditions or states.
3. Working backward from that last stage, solve each intermediate stage.
4. Obtain the optimal solution for the original problem by solving all stages sequentially.

Terminology of Dynamic Programming

1. Stage: a period or a logical sub-problem.
2. State variables: possible beginning situations or conditions of a stage.
3. Decision variables: alternatives or possible decisions that exist at each stage.
4. Decision criterion: a statement concerning the objective of the problem.
5. Optimal policy: a set of decision rules developed as a result of the decision criteria that give optimal decisions for any entering condition at any stage.
6. Transformation: normally, an algebraic statement that reveals the relationship between stages.

- In most applications, dynamic programming obtains solutions by working backward from the end of the problem toward the beginning, thus breaking up a large, unwieldy problem into a series of smaller, more tractable problems

Difference between Dynamic Programming and Linear Programming:

Dynamic Programming	Linear Programming
1. It is a multistage decision making process that spans time intervals. Intervals may consist only of stages in which the problem is solved.	1. It gives a solution that will pertain only to one time period with in given capacity, quantity and cost constraints.
2. It is similar to calculus.	2. It is similar to solving set of simultaneous linear equations.
3. It permits one to determine optimal decisions for future time periods, regardless of any earlier decisions.	3. It requires constant updating of any value obtained in order to reflect the current constraints necessary for an optimal answer.
4. Computation technique is not easy.	4. Computation technique is easier.

UNIT-4

What is Goal Programming? Explain in detail.

Goal programming is an off shoot of multi-objective optimization. This is inter connected to the branch of multi-objective decision analysis, which is also known as multi- criteria decision making.

Concepts of Goal Programming:

- Goal programming was first introduced by Charnes, Copper and Ferguson in 1995.
- It was only in 1961 that the actual name appeared in a text.
- Goal programming is used to widely in planning resource allocation policy analysis and functional management issues.
- The first application of goal programming was done on an engineering application for design and placement of the antennas.
- **Objectives:** are referred to the optimization of the measure of performance of a decision, such as profit maximization or cost minimization.
- **Goals:** state target value i.e the minimum acceptable level of performance of any decision taken by the decision maker.
- **Constraints:**
  Are similar to goals in terms of their mathematical formulation.

Difference between Goal Programming and Linear

Programming:

Goal Programming Model Formulation:

Goal Programming	Linear Programming
Has to handle multiple objective functions and attain optimization in multi-criteria decision making.	Has a single objective function to be optimized, like the profit maximization and cost minimization
Effective analysis for a decision maker in a complex system of competing objectives.	Effective only when decision maker is looking for single objective.
Goal Programming problems use ordinal ranking of goal which is decided on the basis of ordinal ranking of goals.	Linear Programming problems are solved on the basis of cardinal value(exact amount)
Ordinal value, which calculates goal programming, is based on their significance or impact on the organization.	Cardinal value, which is the basis of calculation, is expressed in exact amount, i.e. profit or loss.

Goal Programming Model Formulation

↓

Single goal with multiple sub goals

↓

Ranking and Weighting of unequal multiple goals

↓

Equally ranked multiple goals

UNIT-5

What is Queuing theory?

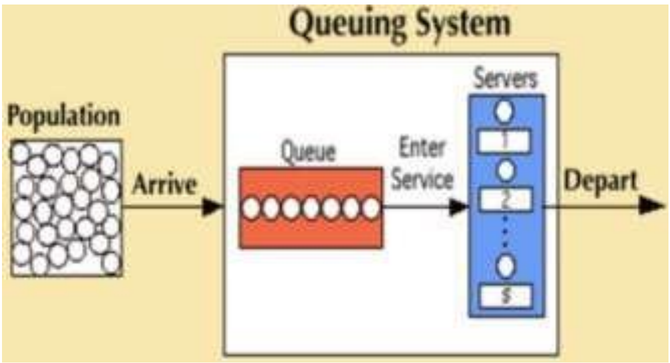
- Queuing theory is the mathematics of waiting lines.
- It is extremely useful in prerding and evaluating system performance.
- Queuing theory has been used for operations research, manufacturing and system analysis.
- Traditional queuing theory problems refer to customers visiting a sotre, analogous to requests arriving at a device.

Applications of Queuing Theory:

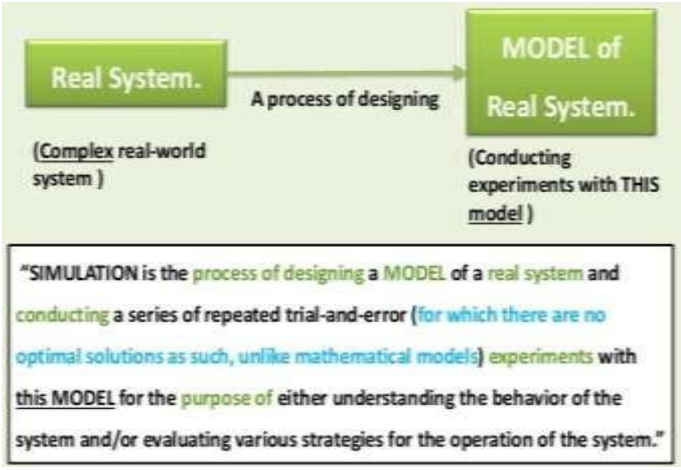
- Telecommunications
- Traffic control
- Determining the sequence of computer
- operations
- Predicting computer performance
- Health services (e.g.. control of hospital bed assignments)
- Airport traffic, airline ticket sales
- Layout of manufacturing systems.

Queuing System:

- Model processes in which customers arrive.
- Wait their turn for service.
- Are serviced and then leave.



What is simulation?



Monte Carlo Simulation:

Statistical simulation technique that provides approximate solution to problems expressed mathematically.
It utilize the sequence of random number to perform the simulation.

- This techniques can be used in different domain
- Complex Integral Computation
- Economics Specially in Risk Management
- extensively used in financial institutions to compute European prices,

- to evaluate sensitivities of portfolios to various parameters and to compute risk measurements

Steps for Monte Carlo Simulation:

Step: 1

- Setting up a probability distribution for variables to be analysed.

Step: 2

- Builiding a cummulative probability distribution for each random variables.

Step: 3

- Genareting the random numbers and then assiging anappropriate set of random numbers to represent value or range. (interval of values for each random variables).

Step: 4

- Conducting the simulation expecteted using ramdom samples.

Step: 5

- Repeat step no 4 until the required no of simulation has been genareted.

Step: 6

- Designing and implementing a course of action and maintaing control.